Research and Computer Implementation of Winning Strategies for Progressively Finite Games

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Description

Progressively Finite Games have a finite number of moves and consist of two players alternating turns until a winner has been declared. Our goal was to determine winning strategies for these types of games. As Alan Tucker explained, "*We can model a progressively finite game by a directed graph with a vertex for each position that can occur in the play of the game and a directed edge for each possible move from one position to another.*"

Mathematical formulas played an important role in devising the winning scenarios for each of the four games: *Reversi*, *Take Away*, *Inverted Take Away*, and *Dots and Boxes*. We utilized Android to demonstrate our solutions.

Before the ACCESO Summer Research Institute began, none of us had any Android Development and any research experience. As the days progressed, our skills in object-oriented design, efficient algorithm design, and app development vastly improved.

REFERENCES:

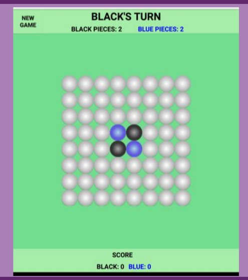
- "Strategy Guide for Reversi & Reversed Reversi" (www.samsoft.org.uk/reversi/strategy.htm)
- en.wikipedia.org/wiki/Reversi
- "Android Application Development - A Beginner's Tutorial" by Budi Kurniawan
- "Android Programming The Big Nerd Ranch Guide" by Bill Phillips & Brian Hardy (http://proquest.safaribooksonline.com/book/programming/android/9780132869126)
- "Applied Combinatorics" – Chapter "Progressively Finite Games" by Alan Tucker
- "Game Theory" by Thomas Ferguson (http://www.math.ucla.edu/~tom/Game_Theory/Contents.html)
- Website to accompany "Game Theory" by Thomas Ferguson (http://www.math.ucla.edu/~tom/Games/dots&boxes.html)
- Elwyn Berlekamp's Game Theory home page (https://math.berkeley.edu/~berlek/cgt/dots.html)
- David Wilson's "Dots-and-Boxes Site" (http://wilson.engr.wisc.edu/boxes/intro.shtml)
- "Winning Ways for Your Mathematical Plays", Vol.3 by Elwyn Berlekamp (http://site.ebrary.com.summit.csuci.edu:2048/lib/csuci/reader.action?docID=10158326)
- Android Developers Website (http://developer.android.com/index.html)

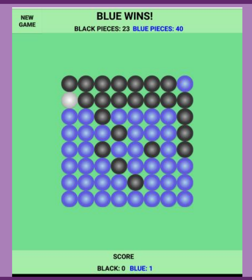
Conclusions

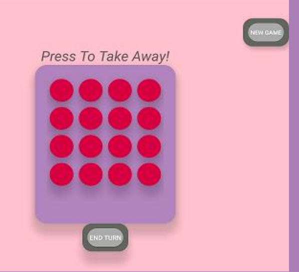
We demonstrated winning algorithms for *progressively finite games* by implementing them on Android platform. It was challenging but very rewarding.

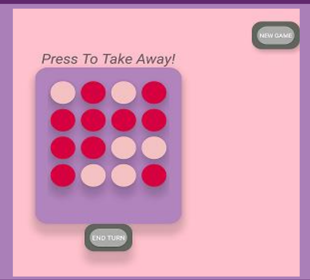
- For *Take Away* and *Inverted Take Away*, we tried every scenario possible by making the first move or second move. In both cases, there was a winning solution, and it depended on who goes first.
- Not all the games have bullet-proof solutions, for example, Reversi invented in 1883 has yet to have a definite winning algorithm.
- Critical thinking and presentation skills were another vital part of the research process. Prior to implementation, as a team, we discussed the variables, methods, objects, and strategies that were going to be utilized for the Artificial Intelligence (AI) portion of our solutions.
- Each game presented a different challenge:
  - *Reversi* – calculating the paths, porting Java solution to Android
  - *Take Away* – translating the formulas to Java
  - *Inverted Take Away* – internal representation of the data using graphs
  - *Dots & Boxes* – understanding different approaches documented in various papers
- Tools also created a challenge, for example porting code from Eclipse to IDEA, learning GUI – layouts, spacing, sizing, placement of objects

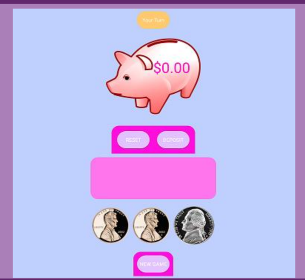
Results

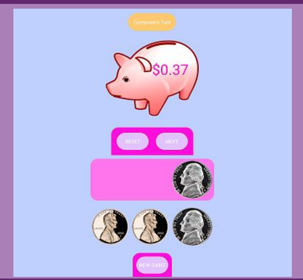


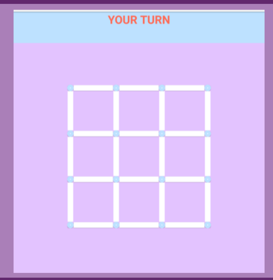


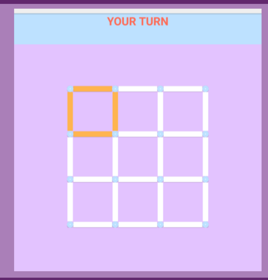






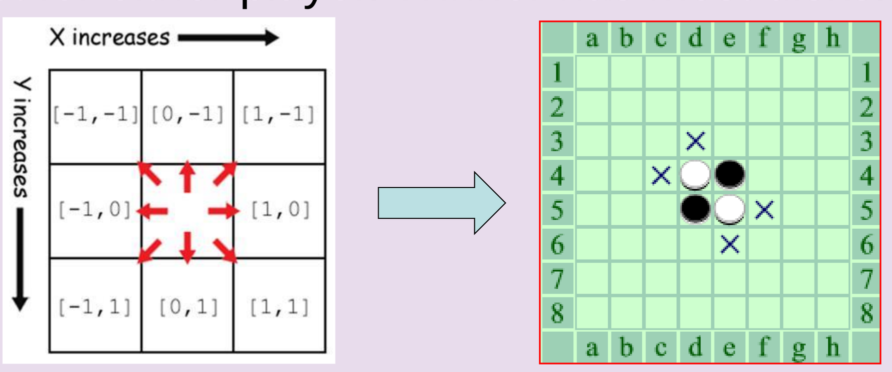






Reversi Game

Objective: Flip opponent's disks by "sandwiching" them. The game ends when no possible moves exist for either player. Winner has most disks.



Technique:

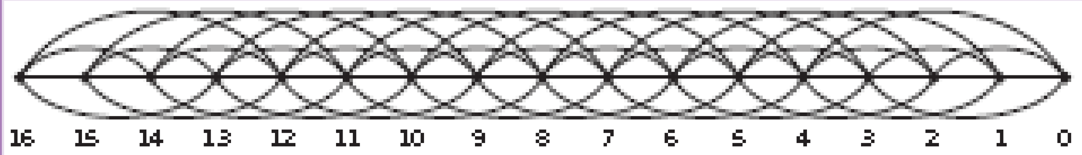
```
if (move has a path in one direction)
     add to collection of possible moves
check other directions
```

Strategy: Get corners; avoid spaces adjacent to corners.

```
if (possible space is a corner) select corner
else if (possible space is near corner) avoid this space
else search for space that will result in most flips
```

Take Away Game

Objective: Two players take turns to take 1 – 4 objects from 16 objects. The player that takes the last object wins.



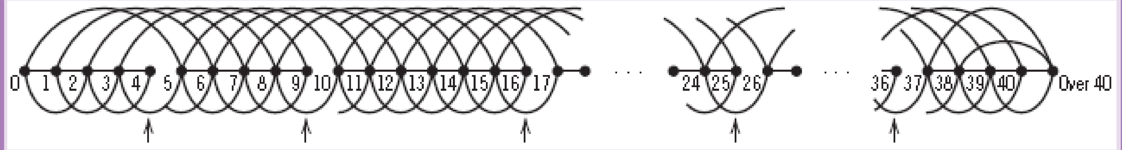
Technique: The winning strategy exists for the **first player.** He is to start by removing one object, and on every subsequent move leave a multiple of five objects (moving to a vertex whose number is a multiple of 5):

```
#objects removed = #remaining objects % 5
```

Whatever the second player's next move is, the first player will always be able to move to the pre-winning vertex 5, and one round later the first player will win.

Inverted Take Away Game

Objective: Starting from 0, reach a number that is either a *square of a number*, or *exceeds 40*. Two players take turns to add 1, 2, or 5 every turn.



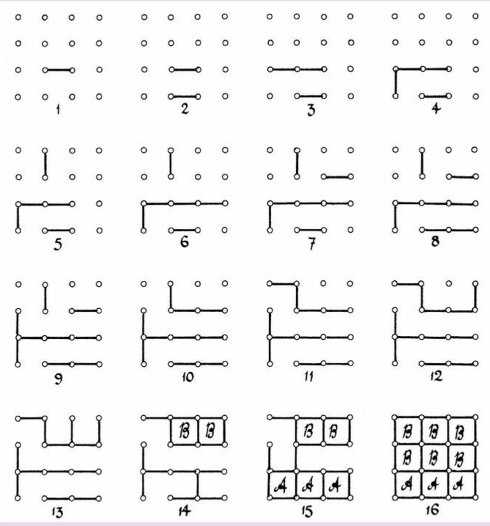
Technique: Build the directed graph with vertices representing all possible moves; see the partial adjacency matrix below:

	[0]	[1]	[2]	[3]	[4]	[5]
[0]		P	2			N
[1]	P		P	2		
[2]	2	P		P	2	
[3]		2	P		P	2
[4]			2	P		P
[5]	N			2	P	

Losing vertices are 2, 3, 7, 8, 11, 14, 15, 20, 23, 24, 31, 34, 35, 37, 38, 39, and 40. The algorithm avoids all these vertices in order to win.

Dots and Boxes Game

Objective: Be the one to acquire more boxes than your opponent. The board starts off filled with dots, each player takes a turn joining two adjacent dots. Upon completing a box, a player must go again.



Technique: We focused on **9-boxes** which has the most defined winning solution:

- First player wants an even number of paths and second player wants an odd number of paths in order to win.
- Each player attempts to force the other player to complete boxes in small chains. Winner wants to complete boxes in large chains.